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Coupled gap equations for the screening masses in hot SU(N) gauge theory

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Abstract. Coupled 1-loop gap equations are studied numerically for non-Abelian electric and magnetic screening in various versions of the three-dimensional effective gauge models. Corrections due to higher dimensional and non-local operators are assessed quantitatively. Comparison with numerical Monte-Carlo investigations suggests that quantitative understanding beyond the qualitative features can be achieved only by going beyond the present treatment.

1 Introduction

An outstanding consequence of heating non-Abelian gauge fields is the screening of static chromo-electric and -magnetic fields. Electric (Debye) screening is generated by non-zero Matsubara modes both for Abelian and non-Abelian gauge fields and its value at leading order has been known since a long time. The dynamics of the non-Abelian zero modes is quite complex [1] and it leads, at least in the Schwinger-Dyson approach to the static magnetic gluon two-point function, to the generation of a magnetic screening mass [2]. Non-zero magnetic mass was obtained from all variants of the magnetic gap equation in the effective three-dimensional gauged Higgs and also in pure gauge theories [2–6].

The spectra of screening masses in the SU(2) Higgs model were also studied in lattice Monte-Carlo simulations by measuring various correlation functions of gauge invariant operators [7–9] as well as the gauge boson propagator in fixed (Landau) gauge [10]. Electric and magnetic screening are simultanously accessible to four-dimensional Monte-Carlo simulations [11]. The magnetic mass obtained from the gauge boson propagator in Landau-gauge is rather close to that obtained from the gap equation, while the gauge invariant gluonic correlation functions yield several times larger mass. A consistent interpretation of the situation emerging in the weakly coupled Higgs+Gauge model was suggested in [12] (see also [13]) in the framework of the so-called constituent model, where the mass measured in fixed gauge correlators corresponds to the constituent magnetic gluon mass of the confined three-dimensional theory. The idea of the constituent gluon offers a natural effective degree of freedom, which dominates the static part of the free energy of the gluon plasma [14]. It should be noticed in this context that the relation between gap equation and the resummation of the free energy has been recently discussed in [15–17] for the scalar field theory.

For pure hot $SU(N)$ theory the situation is more complicated since for the physically interesting range of the temperature the value of the gauge coupling g is close to unity and it is not clear whether the standard argument for dimensional reduction applies.

The aim of the present paper is to investigate how far the screening masses can be described by means of the technique of gap equations. Since the hiearchy of scales $2\pi T >> gT >> g^2T$ fails to hold in pure $SU(N)$ theory for the temperature range of interest we should investigate a coupled set of gap equations for all the screened modes and determine the corresponding screening lengths simultanously.

The most straightforward way to do this would be to derive gap equations in the full four-dimensional theory. However, it is not known how to generalize the by-now well-established three-dimensional gauge invariant resummation [5,6] to four dimensions. There are gauge transformations which might mix static and non-static modes, therefore the resummation of the static modes only, which was suggested in [18] violates gauge invariance. Since screening masses are static quantities it is natural to calculate them in the framework of an effective three-dimensional theory which is, however, valid only up to scales $k \simeq gT$. It was shown that the accuracy of the description of such theories is improved if in the action of the effective theory beside superrenormalizable operators one also takes into account higher dimensional and non-local operators [19]. When deriving the gap equation in the framework of three-dimensional adjoint Higgs model it is important to

address the question whether the symmetric or the broken phase is the physical one. In [7,20] it has been argued that symmetric phase is the physical one, the study of the Debye screening also suggests that the presence of the A_0 condensate is physically unfavourable, therefore in what follows we will assume no A_0 backround.

In this paper we shall assume only the separation of the static and non-static scales in the pure $SU(N)$ gauge theory, that is we assume that $g(T) << 2\pi$. In this way we will be interested in the derivation and the solution of the coupled gap equations of the electric and magnetic static fields. To our best knowledge our paper represents the first attempt for this simple generalisation of the gap-equation approach, where till now electric and magnetic screening masses have been discussed separately. The closest to the spirit of our investigation is the analysis of the sensitivity of higher order corrections of the electric screening mass to the existence and the value of a magnetic screening scale by Rebhan [21].

The paper proceeds as follows: in Sect. 2 we shall calculate screening masses from the local superrenormalizable effective theory (i.e the adjoint Higgs model) of the hot $SU(N)$ gauge theory using a gauge-dependent resummation scheme (namely, R_ξ-gauge). The temperature and gauge-parameter dependence of the results are discussed in detail. Gauge invariant resummation schemes are used for the derivation of similar coupled gap-equations in Sect. 3. In Sect. 4 we shall analyze the effect of higher dimensional and non-local operators in the gap equation. In Sect. 5 detailed comparison with the results of earlier and the most recent numerical investigations will be presented together with our conclusions.

2 Gauge non-invariant resummation scheme

This scheme for the evaluation of the magnetic mass was first suggested in [2] and for the Debye mass in [21]. It was noticed in [2] that magnetic mass obtained in this scheme is gauge dependent and therefore cannot be regarded as physically meaningfull. However, even in gauge invariant resummation schemes the value of the magnetic mass cannot be defined unambigously, because it depends on the specific resummation scheme [22]. Therefore it seems interesting to calculate the magnetic mass in a gauge non-invariant scheme just to compare the amount of gauge dependence with the ambiguity of gauge invariant approaches.

The three-dimensional effective Lagrangian relevant for 1-loop calculations can be written with R_ξ gauge fixing as

$$\begin{aligned} L &= \frac{1}{4}F_{ij}^a F_{ij}^a + \frac{1}{2}\left((D_iA_0)^2 + m_D^2 A_0^a A_0^a\right) \\ &+\frac{1}{2}m_T^2 A_i^a A_i^a + (\partial_i \bar{c}^a D_i c^a + m_G^2 \bar{c}^a c^a) \\ &+\frac{1}{2\xi}(\partial_i A_i)^2 + L_{ct} \ , \\ L_{ct} &= \frac{1}{2}(m_{D0}^2 - m_D^2)A_0^a A_0^a - \frac{1}{2}m_T^2 A_i^a A_i^a - m_G^2 \bar{c}^a c^a \end{aligned} \quad (1)$$

with

$$\begin{aligned} D_i A_0^a &= \partial_i A_0^a - g_3 f^{abc} A_0^b A_i^c \ , \\ F_{ij}^a &= \partial_i A_j^a - \partial_j A_i^a + g_3 f^{abc} A_i^b A_j^c \ . \end{aligned}$$

Here we have added and substracted a mass term for A_i, A_0 and the ghost fields with their exact screening masses. m_{D0}^2 is the tree-level (from the point of view of the effective theory) Debye mass for A_0, which was generated during the procedure of the dimensional reduction. The other parameter of the effective theory is the three-dimensional gauge coupling g_3 which appears in the definitions of the covariant derivatives and the field strength tensor. At 1-loop level one has $m_{D0}^2 = g^2NT^2/3$ and $g_3^2 = g^2T$, where $g = g(T)$ is the gauge coupling of the original theory. The propagators can be read from the quadratic part of the Lagrangian and can be found in the Appendix, where one also finds some details of the evaluation of the relevant Feynman diagrams contributing to the different 2-point functions. It should be noticed when performing the resummation of the pure gauge sector with the unique mass term of mass m_T the longitudinal and transverse gluons acquire different masses, which are, however related by $m_L = \sqrt{\xi}m_T$ (m_T is the transverse and m_L is the longitudinal mass). It is also possible to perform the resummation by introducing independent masses for the longitudinal and transverse gluons, but then the corresponding gap equations will have only complex solutions. The gauge boson self-energy can be decomposed as

$$\begin{aligned} \Pi_{ij}(k) &= (\delta_{ij} - \frac{k_i k_j}{k^2})\Pi_T(k, m_T, m_D, m_G) \\ &+\frac{k_i k_j}{k^2}\Pi_L(k, m_T, m_D, m_G) \ . \end{aligned} \quad (2)$$

In the Appendix we give the expression of Π_{ij} in terms of a few fundamental three-dimensional loop-integrals. These integrals are easily evaluated and an explicit but very cumbersome functional form can be written for the longitudinal and transversal projections of the polarisation matrix.

The self energy for A_0 was first calculated in [21]:

$$\begin{aligned} &\Pi_{00}(k, m_D, m_T) \\ &= m_{D0}^2\frac{g^2N}{4\pi}\Bigg[-m_D - m_T + \frac{2(m_D^2 - k^2 - m_T^2/2)}{k} \\ &\times \arctan\frac{k}{m_D + m_T} + (k^2 + m_D^2)\Bigg(\frac{k^2 + m_D^2}{m_T^2 k} \\ &\times(\arctan\frac{k}{m_D + \sqrt{\xi}m_T} - \arctan\frac{k}{m_D + m_T}) \\ &+(\sqrt{\xi} - 1)\frac{1}{m_T}\Bigg)\Bigg] \ . \end{aligned} \quad (3)$$

The on-shell gap equations now can be written as

$$\begin{aligned} m_T^2 &= \Pi_T(k = im_T, m_T, m_D, m_G) \ , \\ m_D^2 &= \Pi_{00}(k = im_D, m_D, m_T) \ , \\ m_G^2 &= \Pi_G(k = im_G, m_T, m_G) \ . \end{aligned} \quad (4)$$

On the mass-shell $\Pi_{00}(k = im_D, m_D, m_T)$ is gauge parameter independent, but $\Pi_T(k = im_T, m_T, m_D, m_G)$ and

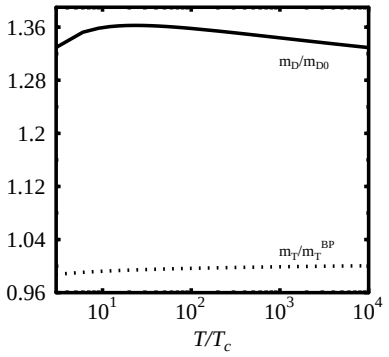

Fig. 1. The temperature dependence of m_D/m_{D0} (*solid line*) and m_T/m_T^{BP} for Feynman ($\xi = 1$) gauge, m_{D0} is the leading order result for the Debye mass and m_T^{BP} is the value of the magnetic mass obtained by Buchmüller and Philipsen for pure $SU(2)$ gauge theory

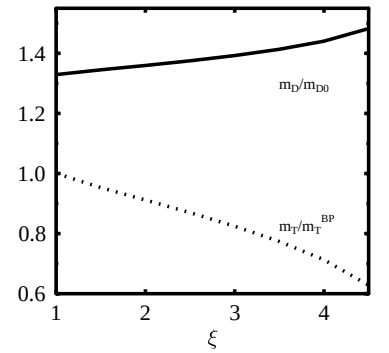

Fig. 2. The dependence of m_D/m_{D0} (*solid line*) and m_T/m_T^{BP} on gauge parameter ξ at $T = 10^4 T_c$, m_{D0} is the leading order result for the Debye mass and m_T^{BP} is the value of the magnetic mass obtained by Buchmüller and Philipsen for pure $SU(2)$ gauge theory

$\Pi_G(k = im_G, m_T, m_G)$ do depend on the gauge fixing parameter, therefore the masses obtained from this coupled set of gap equations are gauge dependent.

In the following numerical investigations we shall consider the case of the $SU(2)$ gauge group. The 4-dimensional coupling constant is taken at scale $\bar{\mu} = 2\pi T$, where $\bar{\mu}$ is the $\overline{MS}$ scale and 1-loop relation for the gauge parameter of the effective theory is used. To set the temperature scale we use the relation $T_c/\Lambda_{\overline{MS}} = 1.06$ obtained from numerical simulation of the finite temperature $SU(2)$ gauge theory [11].

The temperature dependence of m_D in a specific gauge is plotted in Fig. 1. As one can see from the plot m_D receives 30% positive correction compared to the leading order result, while the magnetic mass stays very close to the value calculated by Buchmüller and Philipsen [5], given below in (6). Since the masses are gauge dependent in this approach, it is important to investigate the dependence of the screening masses on the gauge parameter ξ. It turns out that one gets real values for the masses from the gap equations only if $\xi \in [1, 5)$. The dependence of the screening masses on ξ in this range is shown in Fig. 2. One can see, that the ξ-dependence of m_T in this interval is 40%, while for m_D it remains in the 10% range.

3 Gauge invariant approach

Gauge invariant approaches for the magnetic mass generation in three-dimensional pure $SU(N)$ gauge theory were suggested by Buchmüller and Philipsen (BP) [5] and by Alexanian and Nair (AN) [6]. The approach of AN uses the hard thermal loop inspired effective action for the resummation of the magnetic sector. The approach of BP using a gauged σ-model, goes over to the $SU(N)$ gauge theory in the limit of infinitely heavy scalar field. Till now only these two gauge invariant schemes are known to provide real values for the magnetic mass [22].

In these approaches the gauge boson self-energy is automatically transverse and there is no need to project the transverse part from the polarisation tensor. The corresponding expression for the on-shell self-energy reads

$$\Pi_T(k = im_T, m_T) = Cm_T \ , \tag{5}$$

where

$$C = \begin{cases} \frac{g^2 N}{8\pi}[\frac{21}{4}\ln 3 - 1] \ , & AN \ , \\ \frac{g^2 N}{8\pi}[\frac{63}{16}\ln 3 - \frac{3}{4}] \ , & BP \ . \end{cases} \tag{6}$$

Since we are interested in calculating the screening masses in the three-dimensional $SU(N)$ adjoint Higgs model, $\Pi_T(k, m_T)$ should be supplemented by the corresponding contribution coming from A_0 fields. This contribution is calculated from diagrams d) and e) of the Appendix to be

$$\delta\Pi_{ij}^{A_0}(k, m_D) = \frac{g^2 N}{4\pi}\left(-\frac{m_D}{2} + \frac{k^2 + 4m_D^2}{4k}\arctan\frac{k}{2m_D}\right) \times \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \ . \tag{7}$$

It is transverse and gauge parameter independent, it also does not depend on the specific resummation scheme applied to the magnetostatic sector. It should be also noticed that it starts to contribute to the gap equation at $\mathcal{O}(g^5)$ level in the weak coupling regime, thus preserving the magnetic mass scale to be of order $g^2 T$. This is the reason why no "hierarchy" problem arises in this case, at least for moderate g values.

The self energy of A_0 depends on the specific resummation scheme. For BP resummation it reads

$$\begin{aligned} &\Pi_{00}(k, m_D, m_T) \\ &= m_{D0}^2 \frac{g^2 N}{4\pi}\Bigg[-m_D - m_T + \frac{2(m_D^2 - k^2 - m_T^2/2)}{k} \\ &\quad \times \arctan\frac{k}{m_T + m_D} - \frac{(k^2 + m_D^2)}{m_T^2} \\ &\quad \times\left(-m_T + \frac{1}{k}\arctan\frac{k}{m_T + m_D}\right)\Bigg] \ . \end{aligned} \tag{8}$$

This expression is different from the expression of Π_{00} calculated in the gauge non-invariant approach (see (3))

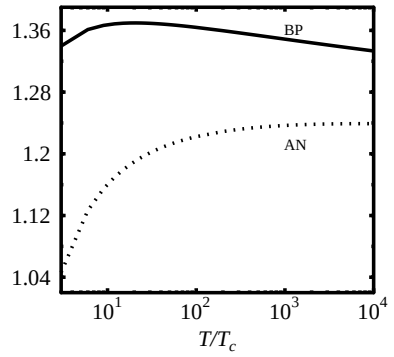

Fig. 3. The temperature dependence of the scaled Debye mass for BP resummation scheme (*solid*) and for the AN resummation scheme (*dashed*). The scaling factor is m_{D0}

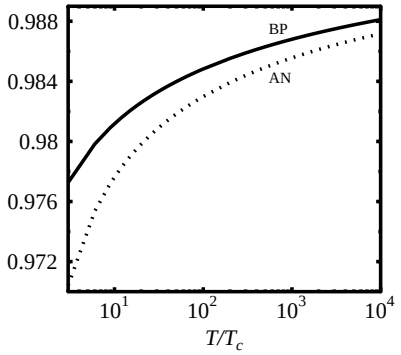

Fig. 4. The temperature dependence of the scaled magnetic mass for BP resummation scheme (*solid*) and for the AN resummation scheme (*dashed*). The scaling factors are m_T^{BP} and m_T^{AN}, respectively

but its analytic properties and on-shell value is the same as of (3). For the resummation scheme of AN the self-energy expression of A_0 coincides with (3) if it is evaluated at $\xi = 1$. The coupled set of gap equations now can be written as

$$m_T^2 = Cm_T + \delta\Pi^{A_0}(k = im_T, m_D) \, ,$$
$$m_D^2 = \Pi_{00}(k = im_D, m_D, m_T) \, . \qquad (9)$$

The temperature dependence of m_D obtained from this coupled set of gap equations is shown in Fig. 3 for both schemes, where we have again normalized the Debye mass by the leading order result, m_{D0}. The temperature dependence of the magnetic mass is shown in Fig. 4, where we have normalized m_T by the value of the magnetic mass obtained for pure three-dimensional $SU(2)$ theory, in the BP (AN) gauge invariant calculations [5,6]. As one can see the contribution of A_0 to the magnetic mass is between 1 and 3%. From Figs. 3 and 4 it is also seen that the temperature dependence of the screening masses is very similar to the temperature dependence of the respective leading order results.

4 Contribution of non-local operators to the gap equation

In the previous section the gap equations were derived for an effective local superrenormalizable theory. In this case the effect of non-static modes in the 2-point function were represented by the thermal mass for A_0 and by the field renormalization factors which relate 3d fields to the corresponding 4d ones. It was shown in [23,24] that when one performs the procedure of the dimensional reduction in R_ξ gauges the parameters of the superrenormalizable effective theory are gauge independent, only the expressions of 3d fields in terms of 4d ones depend on the gauge parameter. However, this does not hold for higher dimensional and non-local operators, which are generally gauge dependent. At 1-loop level the only diagrams contributing non-locally to the gap equation are those which have two non-static line inside the loop, diagrams with one static and one non-static line inside the loop are forbiden because of 4-momentum conservation. Therefore at 1-loop level the non-locality scale $(2\pi T)^{-1}$ is much smaller than the relevant length scales.

In general the non-static contribution to the static 2-point function $\Pi_{\mu\nu}(k_0 = 0, k)$ can be written as

$$\Delta\Pi^{ns}_{\mu\nu}(k_0 = 0, k) = \delta^0_\mu\delta^0_\nu\Pi^{ns}_{00}(k) + \delta^i_\mu\delta^j_\nu(\delta_{ij} - \frac{k_ik_j}{k^2})\Pi(k) \, ,$$
$$\Pi^{ns}_{00}(k) = m^2_{D0} + a_1(\mu,\xi)k^2 + T^2\sum_{n=2}^{\infty} a_n(\xi)(\frac{k^2}{2\pi T})^{2n} \, ,$$
$$\Pi^{ns}(k) = b_1(\mu,\xi)k^2 + T^2\sum_{n=2}^{\infty} b_n(\xi)(\frac{k^2}{2\pi T})^{2n} \, , \qquad (10)$$

where the coefficents a_n and b_n can be calculated for arbitrary n. The first two terms in the expressions of Π_{00} and first term of $\Pi(k)$ are already included into the 3d effective theory as part of the tree level mass and the definition of the 3d fields in term of 4d fields. The last two sums will contribute to the 3d effective lagrangian as quadratic non-local operators. There are also higher dimensional operators as well as non-local 3- and 4-point verticies in the effective lagrangian, however, since we restrict our interest to 1-loop gap equations these are not important for us. Their contribution would correspond to 2 or higher loop contribution in the full four-dimensional theory.

We have estimated by direct numerical evaluation of the infinite sums in (10) the contribution of non-local operators to be less than 1% in the temperature range $T = \left(3 - 10^4\right) T_c$, thus neither their contribution nor their gauge dependence is essential.

5 Conclusion

In the present paper we have made an attempt to extract the electric and magnetic screening masses from the coupled set of gap equations of the three-dimensional $SU(N)$ adjoint Higgs model considered as an effective theory of QCD. The screening masses have been studied using gauge

non-invariant as well as gauge invariant resummation schemes. In the gauge non-invariant formalism we have observed rather strong gauge parameter dependence, therefore the results extracted from it are not very informative. It is still interesting to note that in Feynman gauge ($\xi = 1$) the results for the magnetic mass are rather close to those obtained from the gauge invariant resummation scheme of Buchmüller and Philipsen. In gauge invariant treatments we have compared two different resummation schemes, that of Buchmüller and Philipsen (BP) and one proposed by Alexanian and Nair (AN). Qualitativelly these two resummations lead to similar results, but in the BP scheme one has smaller magnetic mass and larger Debye mass than in the AN scheme.

Let us summarize our view on the interaction of the electric A_0 and the magnetic A_i fields. In both schemes one can see that the dynamics of A_0 is largely influenced by the magnetic sector, however, no similar feedback on the magnetic sector is seen, the magnetic masses calculated from the coupled gap equations provide screening masses which are 1% smaller than evaluated in the pure gauge theory. This fact suggests that the role of the adjoint scalar field is similar to that of the fundamental Higgs field, because the magnetic mass calculated in the symmetric phase of $SU(2)$ Higgs theory using lattice Monte-Carlo simulation with Landau gauge fixing is also roughly the same as in pure gauge theory [10].

Finally we compare our results with recent Monte-Carlo data for the screening masses obtained in 4d finite temperature $SU(2)$ gauge theory [11]. The data on the magnetic mass found from this simulation in the temperature range $T = (10 - 10^4)T_c$ can be fitted well by the formula $m_T = 0.456(6)g^2T$. This value of the magnetic mass is considerably larger than what one obtains from the magnetic gap equation and 3d simulations, where the results are approximately $m_T = 0.28g^2T$ for the BP scheme, $0.38g^2T$ for AN scheme and $0.35g^2T$ for 3d simulation. Also our results are rather close to these values, as we have demonstrated in Sects. 2 and 3.

The data from Monte-Carlo simulation for the Debye mass in the above mentioned temperature interval can be fitted using the following leading order-like anzatz $\sqrt{1.69(2)}g(T)T$ [11], which means that the Debye mass is roughly $1.6m_{D0}$, where m_{D0} is the leading order result. The gap equations at the same time, as one can see from Fig. 3 give $(1.2 - 1.3)m_{D0}$, depending on the resummation scheme. While there is no quantitative agreement between masses measured in Monte-Carlo simulation and those obtained from the gap equation, the temperature dependence of these masses in the temperature interval $T = (10 - 10^4)T_c$ seems to follow the temperature dependence of the leading order result.

At the moment we have no explanation for the reasons of the discrepancy between the results of the 4d simulation and the 3d gap equations and simulation. The observations may imply that either higher loop contributions to the gap equation are important and therefore the dynamics of A_0 is not faithfully reflected by 1-loop gap equations or the dimensional reduction is not valid. In [25] the self-consistency of the gap equation has been examined in the entire momentum space in gauge invariant way. This investigation reveals some inconsistencies of the 1-loop gap equation, which are conjectured to be removed by higher order calculation. It remains an open question how far these inconsistencies influence the pole mass. We believe, however, that further studies in this direction, which include 2-loop gap equations [26] and lattice study of the effective adjoint Higgs model [27] will clarify part of these problems.

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Appendix

The propagators of different fields can be read off the quadratic part of the Lagrangian. These exact propagators are listed below. The gauge boson propagator:

$$D_{ij}(k) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\frac{1}{k^2 + m_T^2} + \frac{k_i k_j}{k^2}\frac{\xi}{k^2 + m_L^2}, \quad (11)$$

where $m_L = \sqrt{\xi} m_T$, the propagator for the adjoint scalar field A_0:

$$D^{A_0}(k) = \frac{1}{k^2 + m_D^2}, \quad (12)$$

and finally, the ghost propagator:

$$\Sigma(k) = \frac{1}{k^2 + m_G^2}. \quad (13)$$

The Feynman diagrams contributing to the 2-point functions of the relevant fields are shown below. In these diagrams every line corresponds to a resummed propagator. The wavy line corresponds to the gauge bosons, the solid one to A_0 and the dashed one to the ghost. The corresponding analytic expressions can be written in terms of the following standard integrals:

$$\begin{aligned} I(k,r,s) &= \int \frac{d^dp}{(2\pi)^d}\frac{1}{(p+k)^{2r}p^{2s}} \\ &= \frac{k^{d-2(r+s)}}{(4\pi)^{d/2}}\frac{\Gamma\left(r+s-\frac{d}{2}\right)}{\Gamma(r)\Gamma(s)}\,\frac{\Gamma\left(\frac{d}{2}-s\right)\Gamma\left(\frac{d}{2}-r\right)}{\Gamma(d-s-r)}\,, \quad (14) \\ I^m(k,r,s) &= \int \frac{d^dp}{(2\pi)^d}\frac{p_m}{(p+k)^{2r}p^{2s}} \\ &= -k^m\,\frac{k^{d-2(r+s)}}{(4\pi)^{d/2}}\frac{\Gamma\left(r+s-\frac{d}{2}\right)}{\Gamma(r)\Gamma(s)} \\ &\quad\times\frac{\Gamma\left(\frac{d}{2}+1-s\right)\Gamma\left(\frac{d}{2}-r\right)}{\Gamma(d+1-s-r)}\,, \quad (15) \end{aligned}$$

$$
\begin{aligned}
&I^{mn}(k,r,s)\\
&=\int\frac{d^dp}{(2\pi)^d}\frac{p^mp^n}{(p+k)^{2r}p^{2s}}\\
&=\frac{k^{d-2(r+s)}}{(4\pi)^{d/2}}\left[\frac{k^2}{2}\frac{\Gamma\left(r+s-1-\frac{d}{2}\right)}{\Gamma(r)\Gamma(s)}\right.\\
&\times\frac{\Gamma\left(\frac{d}{2}+1-s\right)\Gamma\left(\frac{d}{2}+1-r\right)}{\Gamma(d+2-s-r)}\ \delta^{mn}\\
&\left.+\frac{\Gamma\left(r+s-\frac{d}{2}\right)}{\Gamma(r)\Gamma(s)}\frac{\Gamma\left(\frac{d}{2}+2-s\right)\Gamma\left(\frac{d}{2}-r\right)}{\Gamma(d+2-s-r)}\ k^mk^n\right],
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&J(k,m_1,m_2)\\
&=\int\frac{d^3p}{(2\pi)^3}\frac{1}{((p+k)^2+m_1^2)\,(p^2+m_2^2)}=\frac{A}{8\pi}\,,
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&J^m(k,m_1,m_2)\\
&=\int\frac{d^3p}{(2\pi)^3}\frac{p_m}{((p+k)^2+m_1^2)\,(p^2+m_2^2)}\\
&=\frac{k^m}{8\pi}\left[\frac{m_1-m_2}{k^2}-\frac{k^2+m_1^2-m_2^2}{2k^2}\,A\right],
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&J^{mn}(k,m_1,m_2)\\
&=\int\frac{d^3p}{(2\pi)^3}\frac{p^mp^n}{((p+k)^2+m_1^2)\,(p^2+m_2^2)}
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
&=-\frac{\delta^{mn}}{8\pi}\left[\frac{m_1}{2}+\frac{4k^2m_2^2+\left(k^2+m_1^2-m_2^2\right)^2}{8k^2}\,A\right]\\
&-\frac{k^mk^n}{8\pi}\left[\frac{m_1}{2k^2}-\frac{4k^2m_2^2+3\left(k^2+m_1^2-m_2^2\right)^2}{8k^4}\,A\right]\\
&+\frac{1}{8\pi}\frac{(m_1-m_2)\left(k^2+m_1^2-m_2^2\right)}{4k^2}\\
&\times\left[\delta^{mn}-3\,\frac{k^mk^n}{k^2}\right],
\end{aligned}
\tag{20}
$$

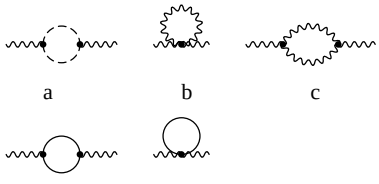
where $A=\frac{2}{k}\arctan\left(\frac{k}{m_1+m_2}\right)$

$$
j(m)=\int\frac{d^3p}{(2\pi)^3}\frac{1}{p^2+m^2}=-\frac{m}{4\pi}\,,
\tag{21}
$$

$$
l^{mn}(m)=\int\frac{d^3p}{(2\pi)^3}\frac{p_mp_n}{p^2+m^2}=-\frac{\delta^{mn}}{3}m^2j(m)\,.
\tag{22}
$$

These integrals were evaluated using dimensional regularization.

The diagrams contributing to the 2-point functions of A_i are the following:



with the analytical contribution:

$$
\begin{aligned}
&\Pi^{(a)\,mn}_{ab}(k)\\
&=g^2N\delta_{ab}\Big[J_{mn}(k,m_G,m_G)+k_mJ_n(k,m_G,m_G)\Big]\,,
\end{aligned}
\tag{23}
$$

$$
\Pi^{(b)\,mn}_{ab}=g^2N\delta_{ab}\delta^{mn}\left[\frac{4}{3}j(m_T)+\frac{2}{3}\xi j(m_L)\right],
\tag{24}
$$

$$
\begin{aligned}
&\Pi^{(c)\,mn}_{ab}(k)\\
&=-\frac{1}{2}g^2N\delta_{ab}\Big\{\delta_{mn}\Big[-\frac{\left(m_T^2+k^2\right)^2}{m_T^2}\\
&\times\left(J(k,m_T,0)+J(k,0,m_T)\right)\\
&+\frac{2k^2\left(4m_T^2+k^2\right)}{m_T^2}J(k,m_T,m_T)\\
&-2\left(1+\frac{k^2}{m_T^2}\right)j(m_T)\Big]\\
&+k_mk_n\Big[-\frac{\left(m_T^2+k^2\right)^2}{4m_T^4}J(k,m_T,0)\\
&+\left(\frac{k^4}{4m_T^4}-\frac{k^2}{m_T^2}-6\right)J(k,m_T,m_T)\\
&+\left(-\frac{k^4}{4m_T^4}+\frac{3}{2}\,\frac{k^2}{m_T^2}+\frac{7}{4}\right)J(k,0,m_T)\\
&+\frac{3}{2m_T^2}j(m_T)+\frac{k^4}{4m_T^4}I(k,1,1)\Big]\\
&+\Big[-\frac{k_mk_n}{m_T^2}j(m_T)+\frac{k^4}{4m_T^4}I_{mn}(k,1,1)\\
&+\left(\frac{k^4}{4m^4}+4\frac{k^2}{m_T^2}+8\right)J_{mn}(k,m_T,m_T)\\
&-\frac{2}{m_T^2}l_{mn}(m_T)-\left(1+\frac{k^2}{m_T^2}\right)^2\\
&\times\left(J_{mn}(k,0,m_T)+J_{mn}(k,m_T,0)\right)\Big]\\
&+\Big[-\frac{k_mk_n}{m_T^2}j(m_T)+\frac{k^4}{m_T^4}k_mI_n(k,1,1)\\
&-\left(\frac{k^2}{m_T^2}+1\right)\left(\frac{k^2}{m_T^2}+3\right)k_mJ_n(k,m_T,0)\\
&+\left(1-\frac{k^4}{m_T^4}\right)k_mJ_n(k,0,m_T)\\
&+\left(\frac{k^4}{m_T^4}+4\frac{k^2}{m_T^2}+8\right)k_mJ_n(k,m_T,m_T)\Big]\Big\}\\
&-g^2N\delta_{ab}\xi\,\Big\{\delta_{mn}\Big[\frac{\left(m_T^2+k^2\right)^2}{m_L^2}(J(k,0,m_T)\\
&-J(k,m_L,m_T))+\frac{\left(m_T^2+m_L^2+k^2\right)}{m_L^2}j(m_L)\Big]\\
&+\frac{k_mk_n}{4m_T^2m_L^2}\Big[\Big(6k^2m_T^2+7m_T^4+2m_T^2m_L^2
\end{aligned}
$$

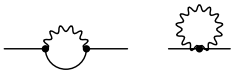
$$
\begin{aligned}
&-\left(m_L^2+k^2\right)^2\Big)J(k,m_L,m_T)\\
&+m_L^2 j(m_T)-7m_T^2 j(m_L)\\
&+\left(k^4-6k^2m_T^2-7m_T^4\right)J(k,0,m_T)\\
&-k^4 I(k,1,1)+\left(m_L^2+k^2\right)^2 J(k,m_L,0)\Big]\\
&+\frac{1}{m_L^2m_T^2}\Big[m_T^2k_mk_nj(m_L)\\
&+k^4\left(J_{mn}(k,m_L,0)-I_{mn}(k,1,1)\right)\\
&+\left(k^2+m_T^2\right)^2\left(J_{mn}(k,0,m_T)\right.\\
&\left.-J_{mn}(k,m_L,m_T)\right)+m_T^2l_{mn}(m_L)\Big]\\
&+\Big[-\frac{\left(k^2+m_T^2\right)\left(k^2-m_T^2+m_L^2\right)}{m_L^2m_T^2}k_mJ_n(k,m_L,m_T)\\
&-\frac{k^4}{m_L^2m_T^2}k_mI_n(k,1,1)+\frac{k_mk_n}{m_L^2}j(m_L)\\
&+\frac{k^2}{m_T^2}\left(\frac{k^2}{m_L^2}+1\right)k_mJ_n(k,m_L,0)\\
&+\frac{k^4-m_T^4}{m_T^2m_L^2}k_mJ_n(k,0,m_T)\Big]\Big\}\\
&-\frac{1}{2}N\delta_{ab}\xi^2\Big\{\frac{k_mk_n}{4m_L^4}\Big[-2m_L^2j(m_L)\\
&+k^4\left(I(k,1,1)+J(k,m_L,m_L)\right)\\
&-\left(k^2+m_L^2\right)^2J(k,m_L,0)-\left(k^2-m_L^2\right)^2J(k,0,m_L)\Big]\\
&+\frac{k^4}{m_L^4}\Big[I_{mn}(k,1,1)-J_{mn}(k,m_L,0)\\
&-J_{mn}(k,0,m_L)+J_{mn}(k,m_L,m_L)\Big]\\
&+\Big[-\frac{k^2}{m_L^2}\left(\frac{k^2}{m_L^2}+1\right)k_mJ_n(k,m_L,0)\\
&-\frac{k^2}{m_L^2}\left(\frac{k^2}{m_L^2}-1\right)k_mJ_n(k,0,m_L)\\
&+\frac{k^4}{m_L^4}\left(k_mJ_n(k,m_L,m_L)+k_nI_m(k,1,1)\right)\Big]\Big\}\,, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
&\Pi^{(d)\,mn}_{ab}(k)\\
&\quad=-\frac{1}{2}g^2N\delta_{ab}\Big[4J^{mn}(k,m_D,m_D)\\
&\qquad+4k^mJ^n(k,m_D,m_D)+k^mk^nJ(k,m_D,m_D)\Big]\,, \qquad (26)
\end{aligned}
$$

$$
\Pi^{(e)\,mn}_{ab}=g^2N\delta_{ab}\delta^{mn}j(m_D)\,. \qquad (27)
$$

Diagrams a), b) and c) were calculated in [28] using the Landau gauge ($\xi=0$). The results of these calculation coincide with ours if in the above formulas one sets $\xi=0$.

The diagrams contributing to the 2-point function of A_0 are given by diagrams f) and g):



f g

The corresponding analytical contribution is:

$$
\begin{aligned}
&\Pi^{(f)\,\mu\nu}_{ab}(k)\\
&\quad=-g^2N\delta_{ab}\delta^{0\mu}\delta^{0\nu}\Big[j(m_T)-\frac{k^2+m_D^2}{m_T^2}j(m_T)\\
&\quad-\frac{\left(m_D^2+k^2\right)^2}{m_T^2}J(k,m_D,0)+\frac{1}{m_T^2}\Big(2k^2\left(m_T^2+m_D^2\right)^2\\
&\quad+k^4+\left(m_T^2-m_D^2\right)^2\Big)J(k,m_D,m_T)-j(m_D)\Big]\\
&\quad-g^2N\delta_{ab}\delta^{0\mu}\delta^{0\nu}\frac{\xi}{m_L^2}\Big[\left(m_L^2+m_D^2+k^2\right)j(m_L)\\
&\quad+\left(m_D^2+k^2\right)^2J(k,m_D,0)\\
&\quad-\left(m_D^2+k^2\right)^2J(k,m_D,m_L)\Big]\,, \qquad (28)
\end{aligned}
$$

$$
\Pi^{(g)\,\mu\nu}_{ab}=g^2N\delta_{ab}\delta^{0\mu}\delta^{0\nu}\Big[2j(m_T)+\xi j(m_L)\Big]\,. \qquad (29)
$$

Finally the single diagram contributing to the ghost 2-point function is given by diagram h):

h

The corresponding analytical contribution is:

$$
\begin{aligned}
\Sigma_{ab}(k)=&-\frac{1}{2}g^2N\delta_{ab}\Big\{\Big[\frac{m_T^2-\left(k^2+m_G^2\right)}{4m_T^2}j(m_T)\\
&-\frac{\left(k^2+m_G^2\right)^2}{4m_T^2}J(k,m_G,0)\\
&+\left(\frac{\left(k^2+m_G^2\right)^2}{4m_T^2}+\frac{k^2-m_G^2}{2}+\frac{m_T^2}{4}\right)\\
&\times J(k,m_G,m_T)-\frac{1}{4}j(m_G)\Big]\\
&-\xi\Big[\left(\frac{\left(k^2+m_G^2\right)^2}{4m_L^2}+\frac{k^2+m_G^2}{2}+\frac{m_L^2}{4}\right)\\
&\times J(k,m_G,m_L)-\frac{1}{4}j(m_G)\\
&-\frac{\left(k^2+m_G^2\right)^2}{4m_L^2}J(k,m_G,0)\\
&-\frac{3m_L^2+k^2+m_G^2}{4m_L^2}j(m_L)\Big]\Big\}\,. \qquad (30)
\end{aligned}
$$

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